Superallowed Fermi Beta Decay and the Unitarity of the Cabibbo-Kobayashi-Maskawa Matrix

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In this work, the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix has been investigated by studying the eleven well-known superallowed Fermi Beta decays; their parent nuclei are ^{10}C, ^{14}O, ^{26}Al, ^{34}Cl, ^{38}K, ^{42}Sc, ^{46}V, ^{50}Mn, ^{54}Co, ^{62}Ga, and ^{74}Rb. The numerical value of the V_{ud} element of the CKM mixing matrix has been calculated following the standart procedure. Using a different method from those of the previous studies, the effect of the isospin breaking due to the Coulomb forces has been evaluated more accurately. Here, the shell model has been modified by Pyatov's restoration because of the isospin breaking and the transition matrix elements have been found by means of the random phase approximation (RPA).

Key words: Superallowed Beta Decays; Isospin Breaking; CKM Matrix.
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1. Introduction

Superallowed $J^\pi 0^+ \to 0^+$ Fermi beta decay in nuclei is a good tool in order to test the results and predictions of the electroweak standard model. Therefore, the superallowed transition is the subject of intense studies for several decades [1 – 14]. One of the most important problems in the standard model of particle physics is the unitarity of the CKM matrix which states the quark eigenstates of weak interaction in terms of quark mass eigenstates. Currently, determination of the matrix elements in order to understand the underlying mechanism of the CP violation in and beyond the standart model is a hot topic in particle physics [15]. There are a few processes to test the unitarity of this matrix in particle and nuclear physics. In the side of nuclear physics the free neutron decay, the pion beta decay, and the superallowed Fermi beta decay are the related processes to consider. The related element of CKM matrix with the superallowed Fermi Beta decay, as is well known, is V_{ud}. According to the unitarity condition [16, 17]:

$$V_{\mathrm{ud}}^2 + V_{\mathrm{us}}^2 + V_{\mathrm{ub}}^2 = 1. \tag{1}$$

Up to date, the contribution from the radiative terms because of the $W^\pm$ gauge bosons mediated theory is understood well [18, 19], the topic of research of this field shifted to the isospin breaking (nuclear mismatch) correction on the transition matrix elements of the model which is used to calculate V_{ud}.

In this field, there are a few active groups which are focused on the investigations of the isospin breaking correction. To date, Towner and Hardy made many calculations for the value of the isospin breaking correction terms using the shell model and full-parantage expansions in terms of Woods-Saxon radial wave functions [2, 4, 5, 14]. Ormand and Brown also used the shell model and Hartree-Fock calculations for the breaking terms in question [6, 10]. Another method based on the formalism of R-matrix theory has been performed by Barker [7, 8]. Next work in which RPA correlations added to a Hartree-Fock calculation that putting together charge symmetry and charge independence belong to Sagawa et al. [11]. Following, a large shell model calculation has been performed for the A = 10 case by Navrátil et al. [12]. Finally, Wilkinson also tried to determine and eliminate the isospin breaking by appropriate extrapolation to $Z \approx 0$ looking on experimental data [9, 13] and using the different particle data to search on the unitarity of the CKM matrix, successively [20 – 24].

In the present work, the isospin breaking due to the isovector part of the shell model potential has been seperated and its effect eliminated by Pyatov's restoration method [25]. Therefore after the restoration, the remaining isospin breaking should be attributed to the Coulomb forces, solely. Here, it is important to emphasize that none of the mentioned publications contact with this major detail.

2. Method

As is well known, the single particle shell model potential is given by

$$U(r) = -U_0 f_0(r) + U_1 f_1(r) t_z + V_C(r). \quad (2)$$

In (2), $f_0(r)$ and $f_1(r)$ are the radial functions of the isoscalar and isovector potentials, U_0 and U_1 are parameters, and $V_C(r)$ is the Coulomb potential, respectively. The form of the Coulomb potential is

$$V_C = \sum_{k=1}^{A} v_C(k) \left(\frac{1}{2} - t_z(k) \right), \quad (3)$$

where

$$t_z = \begin{cases} \frac{1}{2} & \text{for neutrons,} \\ -\frac{1}{2} & \text{for protons.} \end{cases}$$

It is clear that the isovector and Coulomb terms voilate the isospin symmetry of the potential in (2),

$$\lfloor \hat{H}_{sp} - V_C, \hat{T}_\mu \rfloor] \neq 0. \quad (4)$$

Here, in the second quantisation representation single-particle Hamiltonian is

$$\hat{H}_{sp} = \sum_{\tau,j,m} \varepsilon_{jm}(\tau) a^+_{jm}(\tau) a_{jm}(\tau), \quad \tau = n, p, \quad (5)$$

where $\varepsilon_{jm}(\tau)$ is the single-particle energy of the nucleons with the angular momentum j, and $a^+_{jm}(\tau)$, $a_{jm}(\tau)$ is the single-particle creation (annihilation) operator.

The isospin operators $\hat{T}_\mu$ are defined in the following way:

$$\hat{T}_\mu = \begin{cases} \hat{T}_z, & \mu = 0, \\ (\hat{T}_x + \mathrm{i}\mu \hat{T}_y), & \mu = \pm 1. \end{cases} \quad (6)$$

In addition,

$$T_- = \sum_{i=1}^{A} t^i_-, \quad T_+ = \sum_{i=1}^{A} t^i_+.$$

Since the electromagnetic interaction voilates the isospin symmetry, the isospin breaking caused by the Coulomb forces is natural. On the other hand, the isoscalar and isovector parts which represent the strong interaction between the nucleons should satisfy charge independence condition, i. e. the breaking effect of the isovector part should be suppressed using a method. The method which is used in the present study is Pyatov's restoration procedure. According to Pyatov's method, the breaking symmetry of the model hamiltonian is restored by adding a proper residual force. The residual interaction $\hat{h}$ should satisfy the following condition:

$$\lfloor \hat{H}_{sp} - V_C(r) + \hat{h}, \hat{T}_\mu \rfloor = 0. \quad (7)$$

Pyatov showed that $\hat{h}$ has to be in the form of

$$\hat{h} = \frac{1}{2\gamma} \left[\hat{H}_{sp} - V_C(r), \hat{T}_\mu \right]^+ \left[\hat{H}_{sp} - V_C(r), \hat{T}_\mu \right]. \quad (8)$$

γ is an average of double commutator in the ground state,

$$\gamma \equiv C = \left\langle 0 \left[\left[\hat{H}_{sp} - V_C, \hat{T}_{-\mu} \right], \hat{T}_\mu \right] 0 \right\rangle. \quad (9)$$

Such a form of the residual interaction allows us to treat the Coulomb mixing effects of the isospin simply. Thus, the restoration of the isotopic invariance for the nuclear part of the hamiltonian is satisfied, and the total hamiltonian operator can be written in the form

$$\hat{H} = \hat{H}_{sp} + \hat{h}. \quad (10)$$

3. Isobar Analogue States

We shall consider the isobaric 0^+ excitations in odd-odd nuclei generated from the correlated ground state of the parent even-even nuclei by the charge-exchange forces and use the eigenstates of the single particle hamiltonian $\hat{H}_{sp}$ as a basis. The basis set of the particle-hole operators are defined as

$$\hat{A}_j(p,n) = \frac{1}{\sqrt{2j+1}} \sum_m a^+_{jm}(p) a_{jm}(n), \quad (11)$$

$$\hat{A}^+_j(p,n) = \frac{1}{\sqrt{2j+1}} \sum_m a^+_{jm}(n) a_{jm}(p).$$

The bosonic commutation relations of these operators are given by

$$\left\lfloor \hat{A}^+_j(p,n), \hat{A}_{j'}(p,n) \right\rfloor = \delta_{jj'} \left(N_j(n) - N_j(p) \right), \quad (12)$$

$$\left\lfloor \hat{A}_j, \hat{A}^+_{j'} \right\rfloor = -\left\lfloor \hat{A}^+_{j'}, \hat{A}_j \right\rfloor .$$

Here $N_j(n)$ and $N_j(p)$ are the occupation numbers of the corresponding neutron and proton states.

The form of $\hat{h}$ and γ in particle space is given as

$$\hat{h} = \frac{1}{2\gamma}\sum_{j,j'} E_{j'}(n,p)E_j(n,p)\Big(\hat{A}_{j'}(p,n)\hat{A}^+_j(p,n) + \hat{A}^+_{j'}(p,n)\hat{A}_j(p,n)\Big) \tag{13}$$

and

$$\gamma = \sum_j E_j(n,p)\langle j,p||j,n\rangle(N_j(n)-N_j(p)). \tag{14}$$

In these expressions

$$\sum_j E_j(n,p) \equiv \left(\sum_j(\varepsilon_j(n)-\varepsilon_j(p))\langle j,p||j,n\rangle + V_{\text{np}}\right)$$

and

$$V_{\text{np}} \equiv \langle j,p|v_{\text{C}}|j,n\rangle.$$

In RPA, the collective 0^+ states are considered as one phonon excitation described as

$$\hat{Q}^+_i|0\rangle = \sum_j \psi_j(p,n)\hat{A}_j(p,n)|0\rangle. \tag{15}$$

In (15), ψ_j, $\hat{Q}^+_i$ are the real amplitude and the phonon creation operator, respectively. The $|0\rangle$ is the phonon vacuum which corresponds to the ground state of the even-even nucleus,

$$\hat{Q}_i|0\rangle = 0. \tag{16}$$

We obtain the following orthonormalization condition for the amplitudes:

$$\langle 0|\left\lfloor \hat{Q}_i, \hat{Q}^+_i \right\rfloor|0\rangle = \sum_j (N_j(n)-N_j(p))\psi^2_j(p,n). \tag{17}$$

The eigenvalues and the eigenfunctions of the restored hamiltonian can be obtained by solving the equation of motion in RPA,

$$\left\lfloor \hat{H}, \hat{Q}^+_i \right\rfloor|0\rangle = \omega_i\hat{Q}^+_i|0\rangle. \tag{18}$$

Here, the ω_i's are the energies of the isobaric 0^+ states. Employing the conventional procedure of RPA, we obtain the dispersion equation for the excitation energy of the isobaric 0^+ states as

$$\gamma - \sum_j \frac{E^2_j(n,p)(N_j(n)-N_j(p))}{(\omega_i - \varepsilon_j(p,n))} = 0, \tag{19}$$

with $\varepsilon_j(p,n) \equiv (\varepsilon_j(p)-\varepsilon_j(n))$.

The amplitude can be expressed analytically in the following form:

$$\psi_j(p,n) = \frac{E_j(n,p)}{(\omega_i - \varepsilon_j(p,n))}\frac{1}{\sqrt{Z(\omega_i)}}, \tag{20}$$

with

$$Z(\omega_i) = \frac{\sum_j E^2_j(n,p)}{(\omega_i - \varepsilon_j(p,n))^2}(N_j(n)-N_j(p)). \tag{21}$$

4. Fermi Beta Transitions

The isobaric 0^+ states in the neighbour odd-odd nuclei (N-1, Z+1 and N+1, Z-1) are characterized by the Fermi transition matrix elements between these states and the neighbour even-even nuclei. One could obtain the following Fermi transition matrix elements by using the wave functions above:

a) for the transitions (N,Z) $\to$ (N-1,Z+1);

$$\begin{aligned} M^i_{\beta^-} &= \langle 0|\left\lfloor \hat{Q}_i, \hat{T}_- \right\rfloor|0\rangle \\ &= \sum_j \psi_j(p,n)\langle j,p||j,n\rangle(N_j(n)-N_j(p)), \end{aligned} \tag{22}$$

b) for the transitions (N,Z) $\to$ (N+1,Z-1);

$$\begin{aligned} M^i_{\beta^+} &= \langle 0|\left\lfloor \hat{Q}_i, \hat{T}_+ \right\rfloor|0\rangle \\ &= -\sum_j \psi_j(p,n)\langle j,p||j,n\rangle(N_j(n)-N_j(p)). \end{aligned} \tag{23}$$

It is possible to show that the transitions in question obey the Fermi sum rule

$$\begin{aligned} &\sum_i \left\{|M^i_{\beta^-}|^2 - |M^i_{\beta^+}|^2\right\} \\ &= \sum_j |\langle j,p||j,n\rangle|^2(N_j(n)-N_j(p)) \\ &= 2T_0 = N - Z. \end{aligned} \tag{24}$$

The details of the method and the first application to the isospin breaking have been given in [25] and [26], respectively. The recent applications have been made to the rotational invariance in [27] and the isotopic invariance in [28 – 30]. In addition, Pyatov's method has been applied also to other symmetries in nuclear structure physics [31 – 40].

5. The Value of V_{ud}

Superallowed beta decay ($J^\pi = 0^+$, $T = 1 \rightarrow J^\pi = 0^+$, $T = 1$) between the mother and the daughter nuclei is a usuable tool to probe the electroweak interaction. Since only vector current contribute to these transitions, the experimental ft value is related to the vector coupling constant G_V,

$$ft = \frac{K}{G_V^2 |M_F|^2}, \tag{25}$$

with

$$K/(\hbar c)^6 = 2\pi^3 \hbar \ln 2/(m_e c^2)^5 = (8120.271 \pm 0.012) \times 10^{-10} \quad \frac{\mathrm{s}}{\mathrm{GeV}^4}.$$

Nucleus dependent corrections should be obtained from the experimental ft values. Introducing the correction terms the experimental ft values modified by

$$Ft \equiv ft(1 + \delta_R)(1 - \delta_C) = \frac{K}{2G_V^2 (1 + \Delta_R^V)}. \tag{26}$$

Here, f is the statistical rate function, t is the partial half-life for the transition. δ_C, δ_R, and Δ_R^V are the isospin breaking correction, the radiative correction and the nucleus independent radiative correction, respectively.

The radiative correction is separated into two terms,

$$\delta_R = \delta'_R + \delta_{NS}. \tag{27}$$

The first term is independent of the nuclear structure while the second one depends on nuclear structure as does δ_C. Because of this separation the left side of the statement (26) becomes

$$Ft \equiv ft(1 + \delta'_R)(1 + \delta_{NS} - \delta_C). \tag{28}$$

In consequence, in (28), the first correction term is independent of the nuclear structure and the second term is related with the structure. In addition, separation into two terms of δ_C has became a tradition such as

$$\delta_C = \delta_{C1} + \delta_{C2}. \tag{29}$$

Here, δ_{C1} represents the effect of Coulomb and other charge dependent nuclear forces that cause configurations mixing among the 0^+ state wave functions in the parent and also daughter nuclei. The δ_{C2} includes the other effect of Coulomb interaction, i. e. it gives rise to different binding energies because of the different radial wave functions of the decaying proton in the parent nucleus and of the neutron which is transformed by the decay process in the daughter nucleus.

In electroweak theory, the relationship between Fermi and vector coupling constant is $G_V = G_F V_{ud}$. The Fermi coupling constant G_F, is obtained from muon beta decay. From (26) and (28), the matrix element V_{ud} is [11]

$$V_{ud}^2 = \frac{2984.38(6)}{Ft}. \tag{30}$$

In the calculations, the Woods-Saxon potential with the Chepurnov parametrization [41] was used. The basis used in our calculation contains all neutron-proton transitions which change the radial quantum number n by $\Delta n = 0, 1, 2, 3$. Here, the calculations have been performed for eleven well-known superallowed beta transitions.

Conventionally, the value of the isospin breaking correction term δ_{C1} has been calculated by

$$|M_F|^2 = 2(1 - \delta_{C1}). \tag{31}$$

However, as mentioned above the effect of isospin breaking caused by isovector term should be suppressed in this calculation. If this suppression is missing, the obtained results by means of this calculation could not be reliable values. In our best knowledge, up to date, previous publications has not been contacted with this point.

6. Calculations and Results

In order to solve the problem, in the present work, Pyatov's restoration method has been used. The matrix elements have been calculated by means of the restored hamiltonian. Thus, using (31), the value of δ_{C1} which includes only the influence of the Coulomb interaction, has been obtained.

In the Table 1, the calculated δ_{C1} values in previous studies and the present work have been tabulated for comparation. Table 1 corroborates our argument about elimination of the effect of the isovector term in the potential (2). For that reason, the obtained results for δ_{C1} in the present work are quite different from the values of the previous studies.

According to [1] the δ_{C1} must be positive definite. It should be noted from Table 1 that δ_{C1} can take negative values, too. However, Blin-Stoyle did not consider the

Isobars	Present Work δ_{C1} (%)	[6] δ_{C1} (%)	[8][(a)] δ_{C1} (%)	[8][(b)] δ_{C1} (%)	[11] δ_{C} (%)	[14] δ_{C1} (%)
$^{10}C \to {}^{10}B$	1.399	–	0.070	0.008	−0.01	0.010
$^{14}O \to {}^{14}N$	0.578	0.01	0.050	0.038	0.22	0.050
$^{26}Al \to {}^{26}Mg$	0.168	0.01	0.011	−0.002	0.27	0.040
$^{34}Cl \to {}^{34}S$	0.017	0.06	0.049	0.002	0.34	0.105
$^{38}K \to {}^{38}Ar$	−0.150	0.11	0.060	0.271	0.32	0.100
$^{42}Sc \to {}^{42}Ca$	−0.142	0.11	0.012	0.314	0.43	0.060
$^{46}V \to {}^{46}Ti$	−0.219	0.01	0.022	0.006	–	0.095
$^{50}Mn \to {}^{50}Cr$	−0.449	0.004	0.009	0.002	–	0.055
$^{54}Co \to {}^{54}Fe$	−0.245	0.005	0.015	0.011	0.46	0.040
$^{62}Ga \to {}^{62}Zn$	0.716	–	–	–	1.56	0.330
$^{66}As \to {}^{66}Ge$	0.448	–	–	–	0.78	0.250
$^{74}Rb \to {}^{74}Kr$	0.516	–	–	–	0.70	0.130

Table 1. Values of δ_{C1} (%) compared with previous calculations.

[(a)] Values of δ_{C1} (%) using the formalism of R-matrix theory.
[(b)] Values of δ_{C1} (%) using a pure coulomb CD (charge-dependent) interaction.

Parent Nucleus	ft (sec)	δ'_R (%)	δ_{NS} (%)	δ_{C1} (%)	δ_{C2} (%)	Ft (sec)
^{10}C	3039.5(47)	1.679(4)	−0.345(35)	1.399	0.165(15)	3103.0(51)
^{14}O	3042.5(27)	1.543(8)	−0.245(50)	0.578	0.275(15)	3127.7(34)
^{26m}Al	3037.0(11)	1.478(20)	0.005(20)	0.168	0.280(15)	3140.7(19)
^{34}Cl	3050.0(11)	1.443(32)	−0.085(15)	0.017	0.550(45)	3146.4(24)
^{38m}K	3051.1(10)	1.440(39)	−0.100(15)	−0.150	0.550(55)	3152.3(27)
^{42}Sc	3046.4(14)	1.453(47)	0.035(20)	−0.142	0.645(55)	3148.8(30)
^{46}V	3049.6(16)	1.445(54)	−0.035(10)	−0.219	0.545(55)	3155.3(32)
^{50}Mn	3044.4(12)	1.445(62)	−0.040(10)	−0.449	0.610(50)	3155.0(30)
^{54}Co	3047.6(15)	1.443(71)	−0.035(10)	−0.245	0.720(60)	3148.4(35)
^{62}Ga	3075.5(14)	1.459(87)	−0.045(20)	0.716	1.20(20)	3131.4(72)
^{74}Rb	3084.3(80)	1.50(12)	−0.075(30)	0.516	1.50(25)	3137.5(132)
					Average Ft	3140.6(9)

Table 2. Ft values of the eleven superallowed beta transitions. The data for ft, δ'_R (%), δ_{NS} (%), δ_{C2} (%), and $\Delta_R^{\nu} = (2.361 \mp 0.038)\%$ are adopted from [45]. The nucleus independent radiative correction Δ_R^V is included in the Ft values in (28).

sum rule like (24) because his argument is limited only to the shell and Fermi gas model, i. e. it has not been considered the collective interactions. In the present work, the restoration term $\hat{h}$ (8) represents the residual collective forces. In consequence, according to the sum rule, firstly, the matrix elements of the Fermi beta decays could not take arbitrary values, i. e. the sum rule restrict the values of the matrix elements. Secondly, the negative and the positive beta decays could not be considered independently, thus the sum rule connects each other:

$$\sum_i \left\{ \left| M^i_{\beta^-} \right|^2 - \left| M^i_{\beta^+} \right|^2 \right\} = 2T_0$$

hence,

$$\sum_i \left| M^i_{\beta^-} \right|^2 = 2T_0 + \sum_i \left| M^i_{\beta^+} \right|^2 .$$

The sum rule makes possible superallowed Fermi transitions matrix elements larger than $\sqrt{T_0}$ [26], which means δ_{C1} may be negative. As is seen in Table 1, the negative δ_{C1} values have been obtained also in [8] and [11] for several transitions. In [11] has been given only total δ_C ($\delta_C = \delta_{C1} + \delta_{C2}$) however it implies that δ_{C1} is negative for the $^{10}C \to {}^{10}B$ transition. In addition, in [26], δ_{C1} is negative also for the transitions $^{50}Mn \to {}^{50}Cr$ and $^{54}Co \to {}^{54}Fe$.

Table 3. Unitarity ($V_{ud}^2 + V_{us}^2 + V_{ub}^2$) of the CKM matrix.

(V_{ud}) Present Work	(V_{us})	(V_{ub})	The Obtained Unitarity ($V_{ud}^2 + V_{us}^2 + V_{ub}^2$)
0.9748(4)	0.2196(23) from [42]	0.0036(10) from [42]	0.9985(13)
0.9748(4)	0.2234(18) from [23]	0.00361(47) from [23]	1.0001(11)
0.9748(4)	0.2259(18) from [43]	0.00367(47) from [43]	1.0013(11)
0.9748(4)	0.2257(21) from [44]	0.00431(30) from [44]	1.0012(12)

The experimental ft values, δ'_R, δ_{NS}, δ_C, and the Ft values are listed in Table 2. The Ft values are calculated using (28).

The average $\overline{Ft}$ value of the eleven data in Table 2 is

$$\overline{Ft} = 3140.6(9) \text{ sec.}$$

From (30) we obtain

$$V_{ud}^2 = 0.9502(8),$$

which becomes

$$V_{ud} = 0.9748(4).$$

To search on the unitarity, the numerical values of V_{ub} and V_{us} have been adopted from [23, 42–44]. In order to avoid any eclecticsism, the unitarty of the CKM matrix calculated using several different data for V_{us} and V_{ub} elements.

The average value of the obtained results in Table 3 is

$$\overline{V_{ud}^2 + V_{us}^2 + V_{ub}^2} = 1.0003(12).$$

It is noted that the numerical value here for the unitarity condition is close to the result in [45].

It is important to stress here that there is not a unique concensus on the data of these matrix element in particle physics. The different groups that follow different procedures could give different data for them. Discrepancies on the data in the particle physics affect the unitarity examination directly. However, V_{ud} is the biggest one in (1), i. e. the unitarity condition is more sensitive to the value of V_{ud} than the value of others.

As is seen in Table 2 the present results are essentially in good agreement with the conserved vector current (CVC) hypothesis approximately at the level of 0.06% as in [6].

7. Conclusion

Using the different method we obtained different numerical values for the isospin breaking correction. Thus, it is seen that elimination of the effect of the isovector part of the nuclear shell potential by Pyatov's method has a critical role in the calculation of the isospin breaking correction.

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